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RELATIVE CORRESPONDENCE METHOD AND ITS APPLICATION IN MEASUREMENT PRACTICE

V. I. Gudkov and V. P. Motulevich UDC 536.2:536.5

The fundamental possibilities of applying an approximate method of calculating physical processes — the relative correspondence method — to the indirect measurement of quantities are analyzed. Practical examples are given.

The fundamental propositions of the relative correspondence method were formulated in [1, 2]. The effectiveness of its application for approximate calculations of physical processes was illustrated on the solution of a number of practical problems [1-3]. Further analysis of the method showed, however, that the feasibility of its use is not confined to the circle of theoretical—calculation problems but also extends to the region of measurement practice [4].

Quantities are measured indirectly in the majority of cases in engineering and scientific research. The unknown physical quantity y, dependent on several quantities, is defined by the general equation

$$y = i(x_1, x_2, \ldots, x_p) + \beta I(x_1, x_2, \ldots, x_l), \quad (1)$$

where $i(x_1, x_2, \ldots, x_p)$ and $I(x_1, x_2, \ldots, x_l)$ are the measured quantities or functions of these quantities. The expression (1) is written so as to isolate a certain parameter or complex

$$\beta = f(x_1, x_2, \ldots, x_j, \ldots, x_k), \quad (2)$$

the determination of which is difficult under the measurement conditions. The latter may be a consequence of the fact that it is impossible to measure the quantity β directly, while for its calculation or extrapolation there is either an approximate or an exact but very complicated relation. In a number of cases such difficulties prove to be surmountable using the relative correspondence method.

The essence of this method consists in the following: In the determination of relative quantities with a given degree of confidence it is possible to use a less precise model of the process than in the determination of absolute quantities.

Let us initially consider β as a function of one argument x_j,

$$\beta = f = f(x_j). \quad (3)$$

We assume that a simplified model relation

$$\beta \approx \varphi = \varphi(x_j) \quad (4)$$

exists for β and that the initial quantity

$$f_0 = f(x_{j0}) \quad (5)$$

is known sufficiently exactly for the initial value (favorable for the calculated or experimental determination of β) of the argument x_{j0}. Then the approximate value φ^* of the parameter (complex) β is determined by the calculating formula

$$f \approx \varphi^* = f_0 \frac{\varphi}{\varphi_0}, \quad (6)$$

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where

$$\varphi_0 = \varphi(x_{j0}). \tag{7}$$

Here it is assumed that φ and f are one-to-one functions of the real variable not taking a value of zero.

It is obvious that Eq. (6) is preferable to (4) in that region of the parameters where the relative error of the resultant quantity * proves to be smaller than φ. For the same values of the argument this condition can be expressed, using (6), in the form

$$\left|\frac{\bar{\varphi}-\bar{f}}{\bar{f}}\right| < \left|\frac{\varphi-f}{f}\right|, \tag{8}$$

where

$$\bar{f} \equiv f/f_0; \quad \bar{\varphi} \equiv \varphi/\varphi_0. \tag{9}$$

Let us give cases when the relative correspondence method increases the accuracy of the final result in a calculation from Eq. (6).

In the most general case all the possible variants of the reciprocal combination of two functions with $\varphi/f > 0$ can be divided into two groups as a function of the ratios φ/f and $\bar{\varphi}/\bar{f}$:

1) the condition

$$\varphi/f \gtrless 1; \quad \bar{\varphi}/\bar{f} \gtrless 1 \tag{10}$$

is satisfied for the first;

2) the condition

$$\varphi/f \gtrless 1; \quad \bar{\varphi}/\bar{f} \lessgtr 1 \tag{11}$$

is satisfied for the second.

Two functions such that the difference between them grows in absolute value as the argument increases (Fig. 1a),

$$|\varphi - f| > |\varphi_0 - f_0| \quad \text{for} \quad x > x_0, \tag{12}$$

are called diverging functions, and when this difference decreases (Fig. 1b),

$$|\varphi - f| < |\varphi_0 - f_0| \quad \text{for} \quad x > x_0, \tag{13}$$

they are called converging functions.

One can show that the inequalities (12) and (13) are transformed into corresponding inequalities of the following type:

$$(\varphi - f)/(\varphi_0 - f_0) > 1, \tag{14}$$

$$(\varphi - f)/(\varphi_0 - f_0) < 1. \tag{15}$$

An analysis of combinations of two increasing or two decreasing functions, as the most likely to be encountered in practice, showed the following. The condition (10) is satisfied for diverging decreasing functions [$(\varphi - f)/(\varphi_0 - f_0) > 1$, $\bar{\varphi} < 1$, $\bar{f} < 1$] and the condition (11) for converging increasing functions [$(\varphi - f)/(\varphi_0 - f_0) < 1$, $\bar{\varphi} > 1$, $\bar{f} > 1$]. Diverging increasing [$(\varphi - f)/(\varphi_0 - f_0) > 1$, $\bar{\varphi} > 1$, $\bar{f} > 1$] and converging decreasing [$(\varphi - f)/(\varphi_0 - f_0) < 1$, $\bar{\varphi} < 1$, $\bar{f} < 1$] functions satisfy the equalities appearing either in the condition (10) or in the condition (11) in accordance with the expressions

$$\varphi/f > 1; \quad \bar{\varphi}/\bar{f} \gtrless 1 \tag{16}$$

for $(\varphi - f)/(\varphi_0 - f_0) \gtrless \bar{\varphi}$ or

$$\varphi/f < 1; \quad \bar{\varphi}/\bar{f} \lessgtr 1 \tag{17}$$

for $(\varphi - f)/(\varphi_0 - f_0) \gtrless \bar{f}$. Here the functions ($\bar{\varphi} > 1$, $\bar{f} > 1$) for which $\bar{\varphi} > (\varphi - f)/(\varphi_0 - f_0) > 1$, if $\bar{\varphi} < \bar{f}$, $\varphi > f$ (or $\bar{f} > (\varphi - f)/(\varphi_0 - f_0) > 1$, if $\bar{\varphi} > \bar{f}$, $\varphi < f$) can be called weakly diverging increasing, while the functions ($\bar{\varphi} < 1$, $\bar{f} < 1$) for which $\bar{\varphi} < (\varphi - f)/(\varphi_0 - f_0) < 1$, if $\bar{\varphi} > \bar{f}$, $\varphi > f$ (or $\bar{f} < (\varphi - f)/(\varphi_0 - f_0) < 1$, if $\bar{\varphi} < \bar{f}$, $\varphi < f$) can be called weakly converging

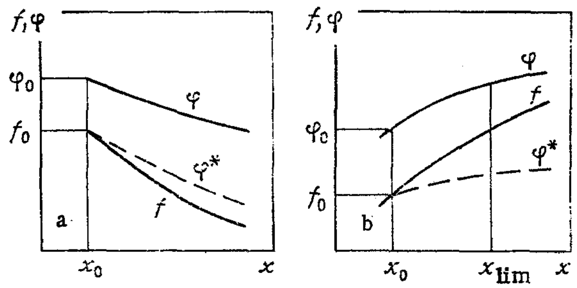

Fig. 1. Scheme of application of the relative correspondence method in the case of diverging decreasing (a) and converging increasing (b) functions φ and f. x_{lim}) limiting value of the argument for the practical application of the method corresponding to Eq. (18).

decreasing functions. As shown in [2], the requirement (8) is always satisfied for the first group of functions (10), while for the second group (11) it is satisfied under the condition

$$\frac{\varphi}{f} \gtreqless \frac{2}{1+f_0/\varphi_0}. \tag{18}$$

In the relations (10), (11), and (16)-(18) it is assumed that the inequalities corresponding to the symbols located on the same level are satisfied.

The method is also preferable in the following particular cases of relation between the functions φ and f.

1. φ and f are proportional to each other: φ = kf, where k = const. Substituting this proportion into (6) and (9), we obtain

$$\varphi^* = f \text{ and } 0 = \frac{\bar{\varphi}-\bar{f}}{\bar{f}} < \left|\frac{\varphi-f}{f}\right| = |k-1|,$$

respectively, i.e., here the method leads to an absolutely exact result.

2. The argument obtains small increments, $\Delta x_j \to 0$. Expanding the functions φ and f in Taylor series in the vicinity of the point x_{j0}, it is easy to ascertain that $\varphi \to \varphi_0$ and $f \to f_0$, and hence

$$0 = \lim_{\Delta x_j \to 0}\left|\frac{\bar{\varphi}-\bar{f}}{\bar{f}}\right| < \lim_{\Delta x_j \to 0}\left|\frac{\varphi-f}{f}\right| = \left|\frac{\varphi_0}{f_0}-1\right|.$$

3. The trivial case when the model function approaches the exact one, $\Delta\varphi \to f$, corresponds to the fact that

$$\lim_{\varphi \to f}\left|\frac{\varphi-f}{f}\right| = \lim_{\varphi \to f}\left|\frac{\bar{\varphi}-\bar{f}}{\bar{f}}\right| = 0.$$

The results of an analysis of a function of one variable presented above can be extended to a function of many variables [2]. Let $f_j = f(x_1, x_2, \ldots, x_j, x_{(j+1)0}, \ldots, x_{k0})$. Then in the obvious equality

$$f = f_0 \frac{f_1}{f_0}\frac{f_2}{f_1}\cdots\frac{f_j}{f_{j-1}}\cdots\frac{f}{f_{k-1}} \tag{19}$$

it is sufficient to substitute the approximate $f_j/f_{j-1} \approx \varphi_j/\varphi_{j-1}$, which follows from the expression (6), to obtain the final calculating formula

$$\varphi^* = f_0 \frac{\varphi_1}{\varphi_0}\frac{\varphi_2}{\varphi_1}\cdots\frac{\varphi_j}{\varphi_{j-1}}\cdots\frac{\varphi_k}{\varphi_{k-1}}, \tag{20}$$

where

$$\varphi_j = \varphi_j(x_1, x_2, \ldots, x_j, x_{(j+1)0}, \ldots, x_{k0}). \tag{21}$$

A graphic illustration of the application of the relative correspondence method to a function of many variables is presented in Fig. 2 in an especially arbitrary plane depiction.

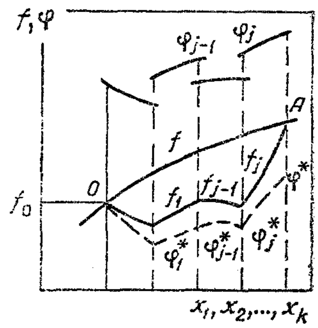

Fig. 2. Arbitrary scheme of application of the relative correspondence method for finding a function of several variables.

The exact function f (which, generally speaking, consists of a certain surface in k-dimensional space) is shown here in the form of a smooth, solid, continuous curve passing through the initial point 0 and the final point A. The solid bent curve is the line along which we will approach the point A if we use Eq. (19). In each continuous segment of this bent curve bounded by vertical lines x_j = const only one argument varies. The series of separate curves of the model functions φ_j corresponding to the exact functions f_j in the isolated segments is shown in the upper part of the figure. And finally, the dashed curve describes the order of determination of the approximate value φ^* using the calculating formula (20).

The method under consideration presumes, at its foundation, the operation with relative quantities. This allows the realization of one more procedure. If when making measurements of physical quantities in practice one uses several sensors similar to each other or several similar modes of operation (constants) of the same sensor, then one can construct the corresponding number of equations of the type (1). For clarity, we consider a variant with two sensors or two constants of one sensor. In this case we have a system of two equations, the solution of which for y has the form

$$y = \{i'(x_1', x_2', \ldots, x_p') I''(x_1'', x_2'', \ldots, x_l'') - i''(x_1'', x_2'', \ldots, x_p'') I'(x_1', x_2', \ldots, x_l') \bar{\beta}\}\{I''(x_1'', x_2'' \ldots, x_l'') - I'(x_1', x_2', \ldots, x_l') \bar{\beta}\}^{-1}, \tag{22}$$

where

$$\bar{\beta} = \frac{f'(x_1', x_2', \ldots, x_j', \ldots, x_k')}{f''(x_1'', x_2'', \ldots, x_j'', \ldots, x_k'')}, \tag{23}$$

while the indices (') and (") refer to the first and second sensors or the first and second state of the sensor, respectively. The ratio of complexes $\bar{\beta}$ of (23), to which the relative correspondence method can be applied, appears in Eq. (22). The calculating relation for the quantity $\bar{\beta}$ follows from the expressions (6), (9), and (23):

$$\bar{\beta} = \frac{f'}{f''} \approx \frac{(\varphi^*)'}{(\varphi^*)''} = \frac{\varphi'}{\varphi''} = \frac{\bar{\varphi}'}{\bar{\varphi}''}. \tag{24}$$

The advantages of such an approach are easy to demonstrate on particular cases when the quantities whose determination is associated with large errors are eliminated from Eqs. (22) and (23).

a) If the complex β remains constant ($f' = f''$) or its variation is negligibly small ($f'' - f' \approx 0$) at all stages of the measurements during the required variation of the arguments, then the unknown parameter y in Eq. (22) does not depend on $\bar{\beta}$, since $\bar{\beta} \approx 1$.

b) By analogy with Sec. 2), the ratio of complexes (23) approaches unity ($\bar{\beta} \to 1$) when the variation of the independent variables is small ($x_1'' \to x_1'$, $x_2'' \to x_2'$, ...). In this case however, one must take into account the increase in the errors in determining small increments of the measured quantities.

c) If the functions f or φ are one-term functions, then individual factors whose values are conserved or vary slightly are cancelled in Eq. (24).

Thus, on the basis of the particular and more general cases analyzed we can conclude that for the functions under consideration the probability of a positive result when using the relative correspondence method in a certain given range of variation of the parameters exceeds 50%.

Let us discuss three concrete examples of measurement of the temperatures of moving heated media.

1. Method of Two Thermometers. When the temperature of a gas stream is measured by contact methods the methodological error is due mainly to the inequality of the intrinsic temperature T of the sensitive element of the sensor and the gas temperature T_m. For thermoelectric sensors in the absence of heat transfer by heat conduction through the conductors of the thermoelectrodes and under conditions of an optically thin gas this temperature difference in a steady state is determined from the equilibrium between convective heat exchange of the hot junction with the gas stream and radiant heat exchange with the surrounding bodies,

$$T_m - T = \frac{\varepsilon\sigma_0}{\alpha}(T^4 - T_{sur}^4). \tag{25}$$

In high-temperature media the fulfillment of the requirement $(T_m - T) \to 0$ is a very difficult task in practice. Therefore, to determine T_m one must also determine α and ε, which introduces additional significant errors.

If one uses two thermocouples with hot junctions of different characteristic sizes d' and d", then one can construct a system of two heat-balance equations (25). We write its solution as [5]

$$T_m = T'\left[1 + \frac{(1-\overline{T})(1-\overline{T}_{sur}^4)}{\overline{\beta}_\alpha \overline{\beta}_\varepsilon(\overline{T}^4 - \overline{T}_{sur}^4) + \overline{T}_{sur}^4 - 1}\right], \tag{26}$$

where $\overline{\beta}_\alpha = \alpha'/\alpha''$, $\overline{\beta}_\varepsilon = \varepsilon''/\varepsilon'$, $\overline{T} = T''/T'$, $\overline{T}_{sur} = T_{sur}/T'$.

The ratio $\overline{\beta}_\alpha$ can be found using one-term approximate functions connecting the Nusselt and Reynolds numbers. Thus, the use of a simplified function of the type Nu = const Re^n for spherical and cylindrical hot junctions of thermocouples leads to the simple expression

$$\overline{\beta}_\alpha = \left(\frac{d''}{d'}\right)^{1-n}. \tag{27}$$

The other ratio $\overline{\beta}_\varepsilon$ for uniform thermocouples is close to unity in practice, and it can be expressed as a linear function of temperature when necessary. If two sensors of the same size but with different and known coefficients ε are used ($\overline{\beta}_\varepsilon \neq 1$), then $\overline{\beta}_\alpha = 1$. The method of two thermometers is successfully used in experimental research, especially when $\overline{T}_{sur} \ll 1$. Let us give a numerical example.

A calculation from Eqs. (26) and (27) with the substitution of the readings of two thermocouples of spherical shape, which are given in [6] (d' = 0.2 mm, d" = 0.5 mm, n = 0.461, $\overline{T}_{sur} \ll 1$), yielded a final result of high accuracy. The discrepancy between the resulting value of T_m and the actual gas temperature did not exceed 0.2% at gas-stream temperatures of 1233 and 1248°K, whereas the relative error of the direct measurements lay in the range of 4-6%.

2. Exponential Method. This method is applicable in the region of high gas-stream temperatures exceeding the melting temperatures of materials. It is based on the measurement of the rise in the temperature of a body placed in the stream in the initial heating period [7]. The differential equation of heating of a body with a small Biot number (Bi < 0.1) and with constant physical properties has a simple analytical solution:

$$\ln\left(\frac{T_m - T}{T_m - T_0}\right) = -m\tau. \tag{28}$$

If we take the ratio of the solutions (28) for two times τ' and τ'', then the heating rate m is eliminated in the result. By solving a system of three Eqs. (28) written for three equally spaced times ($\tau'' - \tau' = \tau''' - \tau''$), we obtain an equation for T_m containing only the temperatures of the sensitive element T', T", and T"' at the chosen times,

$$T_m = T''' \frac{(\bar{T}'')^2 - \bar{T}'}{2\bar{T}'' - \bar{T}' - 1}, \quad (29)$$

where $\bar{T}' = T'/T'''$; $\bar{T}'' = T''/T'''$.

Under the assumption that the heat capacity of the sensor material has a linear dependence on its temperature, $C = C_0 + bT$, where C_0 and b are constants, the calculating equation has the form

$$(T_m - T'')^2 - (T_m - T')(T_m - T''') \exp\left[-\frac{b(2T'' - T' - T''')}{C_0 + bT_m}\right] = 0. \quad (30)$$

The use of this method permitted a two- to threefold decrease in the relative error of measurements of gas temperature in this case. In measurements of the temperatures of a subsonic air stream up to 1000°K the relative error was less than 1.5%, and in temperature measurements up to 3700°K it was about 15% [8].

3. Method of Transitional Modes of Heat Exchange. The method is applicable in principle in high-temperature media. It is based on the solution of the inverse heat-conduction problem, in which the boundary condition is formulated as

$$T_m = T_w + \frac{\lambda}{\alpha}\left(\frac{dT}{dn}\right)_w, \quad (31)$$

where T_w is the surface temperature of the body; $(dT/dn)_w$ is the temperature gradient along the normal. In the simplest variant, considering two different thermal states of a unbounded plane-parallel plate with constant properties, for example, the expression for the indirect determination of the temperature of a moving liquid or gaseous medium will be

$$T_m = T'' \frac{\bar{\beta}_\alpha \bar{T} - \Delta\bar{T}}{\bar{\beta}_\alpha - \Delta\bar{T}}. \quad (32)$$

where $\bar{\beta}_\alpha = \alpha'/\alpha''$; $\bar{T} = T'/T''$; $\Delta\bar{T} = \Delta T'/\Delta T''$.

If $\alpha' = \alpha''$ ($\bar{\beta}_\alpha = 1$), the determination of T_m comes down to the measurement of only two temperature quantities within the plate in two of its temperature states: T' and T", the temperature at any point of the plate, and ΔT' and ΔT", the temperature drops between two points of it.

With a limited relative variation of the coefficient α the functional connection between α and $\bar{\beta}_\alpha$ and the physical parameters can be approximated by simple temperature functions, power-law in particular.

The temperature of a liquid circulating in a constant-temperature bath were measured in [9] by the latter method using cylindrical sensor representing a model of an unbounded plate. With a temperature of 366.8 ± 0.05°K of the medium and a rms error in the temperature measurements within the sensitive element of 0.73%, the relative methodological error of the end result was 1.7% under the assumption that α = const ($\bar{\beta}_\alpha = 1$). In turn, the direct application of Eq. (31) yields a result at least an order of magnitude coarser. This is due to the fact that the absolute value of α can be determined very approximately for the given experiments with unclearly expressed hydrodynamic and temperature conditions at the outer limit of the boundary layer at the sensor surface.

Thus, the data presented provide a basis for stating the following. When making experiments the use of an approximate method of mathematical description of the physical processes taking place with the sensors — the relative correspondence method — makes it possible to considerably simplify the measurements and increase their accuracy in a number of cases.

NOTATION

x, independent (directly measured) quantity; y, dependent (sought) quantity, f, φ, exact and approximate functions describing the physical process; φ*, calculated function determined by the relative correspondence method in accordance with Eqs. (6) and (20); T, temperature; ε, emissivity; σ_0, Stefan—Boltzmann constant; α, coefficient of heat transfer; λ, coefficient of thermal conductivity; C, specific heat; τ, time; m, heating rate. Indices: 0, initial value; m, medium; sur, surrounding bodies; w, sensor wall; ('), pertaining to first sensor or the first state of the sensor; ("), to second sensor or second state of the same sensor; (̄), relative dimensionless quantity or function.

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NONDESTRUCTIVE MONITORING METHODS IN THE INVESTIGATION OF THE THERMOPHYSICAL CHARACTERISTICS OF SOLIDS

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UDC 536.21

Nondestructive methods are proposed for the complex determination of the thermophysical characteristics of solids on the basis of solutions of a system of two-dimensional nonstationary heat-conduction equations. Appropriate computational formulas are presented.

In recent years, a number of papers [1-3] has been published whose authors use the regularities of two- and three-dimensional nonstationary temperature field development in a half-space to determine the thermophysical characteristcs of substances when heat is supplied through a circle of known radius.

If the methods of determining the thermal properties which are based on the two-dimensional nonstationary solutions of the classical boundary-value problem of heat conduction for a half-space are compared with the corresponding one-dimensional methods [4], the deduction can be made that the principal advantage of the former is the possibility of executing complex measurements of the thermal diffusivity, thermal conductivity, and thermal activity coefficients of solids for known values of the temperature and heat flux on just the body surface in its local heating area. Therefore, to find the thermophysical characteristics mentioned from one experiment, there is no need to spoil the wholeness of the specimen and install appropriate sensors therein. Moreover, because of the reduction in the time to prepare the specimen for the experiment, the productivity of the method is raised significantly.

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